

The Destiny of Universes After the Big Trip

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The big trip can be describe with the help of the Wheeler-DeWitt wave equation $\hat{H}\psi(w, a) = 0$. The probability to find the universe after big trip in the state with $w = w_0$ will be maximal if $\partial\psi(w, a)/\partial w|_{w=w_0} = 0$ for any values of the scale factor a . It is shown that this will be the case if and only if $w_0 = -1/3$. This fact allows one to suggest that vast majority of universes in multiverse must be in this state after their big trips.

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I. INTRODUCTION

The cosmology nowadays is amazingly abundant with a new startling solutions. Some of the most recent ones are the models with the "Phantom fields" which result in the violation of the weak energy condition (WEC) $\rho > 0$, $\rho + p/c^2 > 0$ [1], [2], where ρ is the fluid density and p is the pressure. Such phantom fields, as follows from their quantum theory [3], should inevitably be described by the scalar field with the negative kinetic term. The through investigations shows that such fields are apparently could not be considered as a fundamental objects. However, it is possible that the Lagrangians with the negative kinetic terms will appear as some kind of effective models, as it happens in some models of supergravity [4], in the gravity theories with highest derivatives [5] and in field string theory [6], for example in model which is close to the fermion NSR-string with regard for (GSO)-sector (see also [7]). Finally, the "phantom energy" in the brane theory was considered in [8], [9].

Remark 1. Despite of all said above, We can not be assured that phantom energy is only effective model. The reason is the existence of "the crossing of the phantom divide line". There are exact solutions of Einstein equations which describes this crossing. Moreover, this crossing is smooth and one can conclude that smooth (de)-phantomization is the sufficiently general property of Einstein equations. This effect was discover in [10], [11] and it was interpreted in [12]. Therefore if we were (following [12]) guided by a belief that Einstein equations are more fundamental then the concrete form of the Lagrangian for other fields then one can conclude that "the crossing of the phantom divide line" is possible and this is the new fundamental property of gravitation.

The particular interest to models with the phantom fields is caused by their prediction of so-called "Cosmic Doomsday" alias big rip [1] (see also [13]). In case of the phantom energy we have $w = p/(c^2\rho) = -1 - \epsilon$ with

$\epsilon > 0$. Integration of the Einstein-Friedmann equation for the flat universe results in

$$\begin{aligned} a(t) &= \frac{a_0}{(1 - \xi t)^{2/3\epsilon}}, \\ \rho(t) &= \rho_0 \left(\frac{a(t)}{a_0} \right)^{3\epsilon} = \frac{\rho_0}{(1 - \xi t)^2}, \end{aligned} \quad (1)$$

where $\xi = \epsilon\sqrt{6\pi G\rho_0}$. We choose $t = 0$ as the present time, $a_0 \sim 10^{28}$ cm and ρ_0 to be the present values of the scale factor and the density. There, if $t = t_* = 1/\xi$, we automatically get the big rip.

Remark 2. There are few ways to escape of future big rip singularity: (i) to consider phantom energy just as some effective models (see above); (ii) to use quantum effects to delay the singularity [14]; (iii) to use new time variable such that the big rip singularity will be point at infinity ($t \rightarrow \infty$) [15]; (iv) to avoid big rip via another cosmological "Big": big trip (see below).

In [16] Pedro F. González-Díaz had shown that phantom energy can results in achronal cosmic future where the wormholes become infinite before the occurrence of the big rip singularity. To show this lets consider the wormhole with the throat radius $b_0 = 10^{-33}$ cm (Planck scale). It was shown in [16] that if $p = -(1 + \epsilon)c^2\rho$ is a fluid's equation of state, then

$$\dot{c}b(t) = 2\pi^2\epsilon G D \rho(t) b^2(t), \quad (2)$$

where $b(t)$ is the throat radius of a Morris-Thorne wormhole and D is dimensionless quantity. According to [16] we can choose $D \sim 4$ (see also [17]). The equation (2) describes the changing of the $b(t)$ with regard to the phantom energy's accretion. Integration of the (2) gets us

$$\frac{1}{b(t)} = \frac{1}{b_0} - \frac{2\pi^2\epsilon\rho_0 G D t}{c(1 - \xi t)}. \quad (3)$$

Therefore at

$$\tilde{t} = \frac{c}{\epsilon(c\sqrt{6\pi G\rho_0} + 2\pi^2\rho_0 b_0 G D)} \quad (4)$$

we get $b(\tilde{t}) = \infty$. As we can see $\tilde{t} < t_*$, and therefore this universe indeed will be achronal before the occurance of

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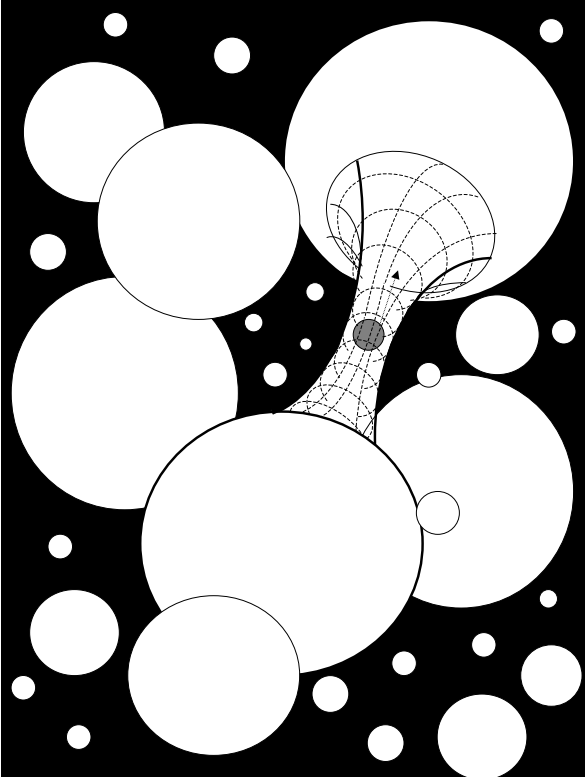


FIG. 1: Pictorial representation of the big trip process when it is carried out by a single grown-up wormhole within the framework of a multiverse picture. In this case the universe does not travel along its own time but behaves like though if its whole content were transferred from one different larger universe to another, also larger universe.

the big rip. In accord to [16], at $t > \tilde{t}$, while in process of the phantom energy's accretion, the wormhole becomes an Einstein-Rosen bridge which can, in principle, be used to escape from the big rip.

Remark 3. In [18] the capability of a phenomenon of big trip was subjected a critic. In response article [19] the detailed answers to all objections of Faraoni were given.

II. BIG TRIP AND WHEELER-DEWITT EQUATION

The big trip is a cosmological process thought to occur in the future by which the entire universe would be engulfed inside a gigantic wormhole and might travel through it along space and time (see Fig. 1).

In this article we'd like to present the possibility of new cosmological "Big" - so called "Big Meeting".

Let us consider the spacetime manifold M for a flat FRW universe with metric:

$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2, \quad (5)$$

where N is the lapse function and $a(t)$ is a scale factor. Using the approach from the [20] one suggest that

the parameter of equation of state $w = p/\rho$ ($c = 1$) is time-dependent one. We don't restrict ourselves by the condition $\dot{w} = 0$ but suppose that $\rho = \rho(w, a)$. If this is the case then differentiating the Einstein-Friedmann equation $H^2 = 8\pi G\rho/3$ with respect to t one get new expression for the scalar curvature \hat{R} :

$$\hat{R} = R + \frac{3}{\dot{a}a^2} (3(1+w)\dot{a}^3 + a^3\dot{\rho}), \quad (6)$$

where

$$R = \frac{6(\dot{a}^2 + a\ddot{a})}{a^2},$$

$$\dot{\rho} = \frac{\partial \rho}{\partial a} \dot{a} + \frac{\partial \rho}{\partial w} \dot{w}.$$

We consider the universe filled with scalar field ϕ with Lagrangian

$$L = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) = p = w\rho.$$

Therefore the action integral of the manifold M with boundary ∂M has the form

$$S = \int_M d^4x \sqrt{-g} \left(\frac{\hat{R}}{16\pi G} + w\rho \right) - \frac{1}{8\pi G} \int_{\partial M} d^3x \sqrt{-h} \text{Tr} \hat{K}, \quad (7)$$

where \hat{R} is the generalized Ricci curvature scalar (6), K is the conventional expression for the extrinsic curvature, $g = \det g_{\mu\nu}$, h is the determinant of the general threemetric on the given hypersurface at the boundary ∂M . Since $\sqrt{-g} \sim Na^3$ then one can integrate (7) over spatial variables and substitute $t \rightarrow i\tau$ (with $8\pi G/3 = 1$) to reduce (7) to Euclidean action:

$$I = \int d\tau N \left[\frac{6aa'^3 - 3aF(a, a', w, w')}{a'N^2} + 6a^3w\rho \right], \quad (8)$$

where

$$F(a, a', w, w') = 3(1+w)a'^3 - a^3 \left(\frac{\partial \rho}{\partial a} a' + \frac{\partial \rho}{\partial w} w' \right),$$

and $' = d/d\tau$. In the gauge where $N = 1$ we have the Hamiltonian constraint

$$H = \frac{\delta I}{\delta N} - (w+1)a^3\rho = 0. \quad (9)$$

The next step to the Wheeler-DeWitt equation is to define the momenta conjugate to a (π_a) and w (π_w) and redefine the (classical) Hamiltonian (9) via π_a , π_w , a and w . At last one must introduce the following quantum operators:

$$\hat{\pi}_a = -i \frac{\partial}{\partial a}, \quad \hat{\pi}_w = -i \frac{\partial}{\partial w},$$

which allows one to obtain the the Wheeler-DeWitt equation (WDE) $\hat{H}\psi = 0$. Thus we have

$$4 \left(a \frac{\partial \rho}{\partial a} + 2\rho \right) \frac{\partial^2 \psi}{\partial w^2} - 4a \frac{\partial \rho}{\partial w} \frac{\partial^2 \psi}{\partial a \partial w} = 3 \left(\frac{\partial \rho}{\partial w} \right)^2 a^6 (1+3w) \psi, \quad (10)$$

with $\psi = \psi(a, w)$.

The WDE (10) is differ from the WDE which was obtained in [20] because we didn't use the condition $\ddot{w} = 0$ which allows one to simplify the actions (7), (8) using the rejection of corresponding surface terms. As we shall see, the Eq. (10) result in new "Big" in cosmology - the "Big Meeting".

First at all, let consider the case $w = -1/3$. In this case the right side of the (10) will be zero. Moreover, if $w = -1/3$ then $\rho = \rho(a, w = -1/3) \sim a^{-2}$ therefore

$$a \frac{\partial \rho}{\partial a} + 2\rho = 0,$$

and the WDE (10) is reduced to

$$\frac{\log a}{a} \frac{\partial^2 \psi}{\partial a \partial w} = 0. \quad (11)$$

Using power series

$$\psi(a, w) = \psi(a, -1/3) + \sum_{n=1}^{\infty} \frac{1}{n!} c_n(a) \left(w + \frac{1}{3} \right)^n,$$

and (11) we get $dc_1(a)/da = 0$ thus

$$\frac{\partial \psi(a, w)}{\partial w} \Big|_{w=-1/3} = c_1 = \text{const.} \quad (12)$$

On the other hand, the equation (11) is invariant with respect to transformation

$$\psi(a, w) \rightarrow \psi(a, w) - f_1(a) - f_2(w), \quad (13)$$

for arbitrary function $f_{1,2}$. Substituting (13) into the (12) and choosing $df_1(w)/dw = c_1$ at $w = -1/3$ we get without loss of generality

$$\frac{\partial \psi(a, w)}{\partial w} \Big|_{w=-1/3} = 0. \quad (14)$$

Therefore, in the case of general position, for any **fixed** a the function $\psi(a, w) = \Phi_a(w)$ has the extremum at $w = -1/3$. Since the function ψ must be normalizable one, the point $w = -1/3$ must be the point of maximum. In other words, the probability distribution $|\psi(a, w)|^2$ for any given value of the scale factor has the peak at $w = -1/3$.

One can prove this fact for the general position. Let consider the equation (10). We'd like to consider the solutions of this equation such that

$$\frac{\partial \psi(a, w)}{\partial w} \Big|_{w=w_0} = 0, \quad (15)$$

for any given a . Since $w_0 = \text{const}$ then $\rho(a, w_0) = \rho_0 = C^2 a^{-3(w_0+1)}$ ($C = \text{const}$) and

$$a \frac{\partial \rho_0}{\partial a} = -3(w_0 + 1)\rho_0, \quad (16)$$

$$\frac{\partial \rho_0}{\partial w_0} = -3 \log a \rho_0. \quad (17)$$

Besides

$$\frac{\partial^2 \psi}{\partial a \partial w} \Big|_{w=w_0} = \frac{\partial}{\partial a} \left(\frac{\partial \psi(a, w)}{\partial w} \Big|_{w=w_0} \right) = 0. \quad (18)$$

Substituting (16), (17) and (18) into the (10) we get

$$(1 + 3w_0) \left(4a^{(-3(w_0+1))} \frac{\partial^2 \psi}{\partial w_0^2} + 27C^2 a^{-6w_0} \log^2 a \psi \right) = 0. \quad (19)$$

Using (19) one can conclude that $w_0 = -1/3$ or the following equation must be hold

$$\frac{\partial^2 \psi}{\partial w_0^2} = -\frac{27C^2}{4} a^{-3(w_0-1)} \log^2 a \psi. \quad (20)$$

The general solution of the (20) has the form

$$\psi(w_0, a) = c_1 J_0(z) + c_2 Y_0(z), \quad (21)$$

where $c_{1,2}$ are arbitrary constants, $z = \sqrt{3}C a^{3(1-w_0)/2}$, J_0 and Y_0 are the Bessel functions of the first and second kind. This function must be normalizable one:

$$\int_0^\infty da |\psi(w_0, a)|^2 < +\infty, \quad (22)$$

so one need to choose $c_2 = 0$ ($Y_0(z) \sim 2 \log z / \pi$ at $z \rightarrow 0$ so if $c_2 \neq 0$ then we get the divergence in the (22) for $a \rightarrow 0$). Substituting (21) into the (18) one get

$$\frac{\partial \psi}{\partial w_0} = \frac{3\sqrt{3}C}{2} a^{3(1-w_0)/2} \log a J_1 \left(\sqrt{3}C a^{3(1-w_0)/2} \right) = 0,$$

which will be the case for arbitrary a if and only if $w_0 = 1$ and C is the solution of the equation

$$C J_1(\sqrt{3}C) = 0.$$

But if $w_0 = 1$ then the wave function $\psi = \text{const}$ (and the same will be the case for the density ρ (see (10))). Such wave function will be non-normalizable one, thus, there is only one way to comply with (15) in framework of normalizable wave function of the universe: to put $w_0 = -1/3$. *The end of the proof.*

One can ask about possibility of existence of another peaks $M_* = (a_*, w_*)$ of solutions of the (10), such that

$$\frac{\partial \psi}{\partial a} \Big|_{M_*} = \frac{\partial \psi}{\partial w} \Big|_{M_*} = 0,$$

and

$$\Delta = \left[\frac{\partial^2 \psi}{\partial w^2} \frac{\partial^2 \psi}{\partial a^2} - \left(\frac{\partial^2 \psi}{\partial w \partial a} \right)^2 \right]_{M_*} > 0. \quad (23)$$

As we shall see, it is possible to neglect these extremum. In fact, one need to use the dimensionless variables in expressions like $\log a$. Keeping in mind that our universe was born via big trip from another, a paternal universe with the value of it's scale factor L , one must to replace $\log a \rightarrow \log(a/L)$. On the other hand, one can expect that the initial value of the scale factor a_i of the universe after the big trip will be $a_i \sim L$. Let consider such universe with $w = \text{const}$. If we'd like to estimate the probability to find this universe just after the big trip (that is with $a_i \sim L \sim a_*$) then one must use the WDE (10) which reduce to the simple form

$$-4a^{-3(w+1)}(3w+1)\frac{\partial^2 \psi}{\partial w^2}|_{M_*} = 0.$$

If $w \neq -1/3$ then (see (23))

$$\Delta = - \left(\frac{\partial^2 \psi}{\partial w \partial a} \right)_{M_*}^2 < 0,$$

and we have not extremum at all.

III. CONCLUSION

Therefore, as we seen, in the case of general position the solution of (10) and the probability distribution has the peaks at $w = -1/3$ for any a if we consider universes just after big trip. This fact result in new possible "Big" in modern cosmology which can be called as "Big Meeting". It means that vast majority of universes in multiverse must be in the state with $w = -1/3$ just after their big trips.

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